PHYSICAL REVIEW B **78**, 220401(R) (2008)

Almeida-Thouless transition below six dimensions

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(Received 22 September 2008; published 2 December 2008)

The existence of an Almeida-Thouless (AT) instability surface below the upper critical six dimensions is demonstrated in the generic replica symmetric field theory. Renormalization flows from around the zero-field fixed point are investigated. By introducing the temperature and magnetic-field dependence of the bare parameters, the fate of the AT line can be followed from mean field ($d=\infty$) down to $d=6-\epsilon$.

DOI: 10.1103/PhysRevB.78.220401 PACS number(s): 75.10.Nr, 03.50.Kk, 05.10.Cc

Notwithstanding the relative simplicity of the relevant model postulated by Edwards and Anderson,[1] the Ising spin-glass problem has resisted a thorough understanding for decades. Severe frustration makes numerical simulations extremely hard and computer time consuming, whereas analytical methods must handle the inhomogenities caused by the quenched disorder. The model was later extended and studied on the fully connected lattice by Sherrington and Kirkpatrick (SK) (Ref. 2); the characterization of the spin-glass phase by the solution of Parisi (see Ref. 3 for a list of references) is now unanimously accepted as the true mean-field theory. This mean-field spin glass proved to be very complicated; its equilibrium state breaks up to ultrametrically organized ergodic components, commonly called pure states. This complex phase-space structure survives in an external magnetic field up to a phase boundary called the Almeida-Thouless (AT) line. Approaching this line from the paramagnetic side an instability develops: using a replicated picture,[1] the diverging spin-glass susceptibility signals the breakdown of the replica symmetric phase,[4] and replica symmetry breaking develops.

An alternative theory—the so-called droplet picture—emerged, however, and continues questioning the relevance of mean-field ideas in finite-dimensional systems.[5] In this theory the glassy phase is much simpler and is limited to zero field: a convincing conclusion about the existence or lack of an AT line may resolve a decade long debate about the structure of the spin-glass phase in the physical dimensions. Recent numerical simulations[6,7] in three dimensions essentially excluded the possibility of a transition in a field, whereas the four-dimensional case remains somewhat ambiguous (see Ref. 8 for references to earlier works). On the analytical side, we must mention the scaling considerations in Ref. 9 and renormalization-group (RG) calculations,[10–12] whereas a leading-order field theoretical computation[13] provided an AT line above six dimensions. This Rapid Communication tries to dissolve the misbelief that the AT line disappears below the upper critical dimension by explicitly calculating it close to but below $d=6$.

Ising spin-glass transition in an external magnetic field can be studied in the generic replica symmetric field theoretical model[14] defined by the Lagrangian $\mathcal{L}=\mathcal{L}^{(2)}+\mathcal{L}^{I}$, where

$$\mathcal{L}^{(2)}=\frac{1}{2}\sum_{\mathbf{p}}\left[\left(\frac{1}{2}p^2+m_1\right)\sum_{\alpha\beta}\phi_{\mathbf{p}}^{\alpha\beta}\phi_{-\mathbf{p}}^{\alpha\beta}+m_2\sum_{\alpha\beta\gamma}\phi_{\mathbf{p}}^{\alpha\gamma}\phi_{-\mathbf{p}}^{\beta\gamma}+m_3\sum_{\alpha\beta\gamma\delta}\phi_{\mathbf{p}}^{\alpha\beta}\phi_{-\mathbf{p}}^{\gamma\delta}\right] \quad (1)$$

and

$$\begin{aligned}\mathcal{L}^{I}=&-N^{1/2}h\sum_{\alpha\beta}\phi_{\mathbf{p}=0}^{\alpha\beta}-\frac{1}{6\sqrt{N}}\sum_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3}{}'\\&\times\Bigg[w_1\sum_{\alpha\beta\gamma}\phi_{\mathbf{p}_1}^{\alpha\beta}\phi_{\mathbf{p}_2}^{\beta\gamma}\phi_{\mathbf{p}_3}^{\gamma\alpha}+w_2\sum_{\alpha\beta}\phi_{\mathbf{p}_1}^{\alpha\beta}\phi_{\mathbf{p}_2}^{\alpha\beta}\phi_{\mathbf{p}_3}^{\alpha\beta}\\&+w_3\sum_{\alpha\beta\gamma}\phi_{\mathbf{p}_1}^{\alpha\beta}\phi_{\mathbf{p}_2}^{\alpha\beta}\phi_{\mathbf{p}_3}^{\alpha\gamma}+w_4\sum_{\alpha\beta\gamma\delta}\phi_{\mathbf{p}_1}^{\alpha\beta}\phi_{\mathbf{p}_2}^{\alpha\beta}\phi_{\mathbf{p}_3}^{\gamma\delta}\\&+w_5\sum_{\alpha\beta\gamma\delta}\phi_{\mathbf{p}_1}^{\alpha\beta}\phi_{\mathbf{p}_2}^{\alpha\gamma}\phi_{\mathbf{p}_3}^{\beta\delta}+w_6\sum_{\alpha\beta\gamma\delta}\phi_{\mathbf{p}_1}^{\alpha\beta}\phi_{\mathbf{p}_2}^{\alpha\gamma}\phi_{\mathbf{p}_3}^{\alpha\delta}\\&+w_7\sum_{\alpha\beta\gamma\delta\mu}\phi_{\mathbf{p}_1}^{\alpha\gamma}\phi_{\mathbf{p}_2}^{\beta\gamma}\phi_{\mathbf{p}_3}^{\delta\mu}+w_8\sum_{\alpha\beta\gamma\delta\mu\nu}\phi_{\mathbf{p}_1}^{\alpha\beta}\phi_{\mathbf{p}_2}^{\gamma\delta}\phi_{\mathbf{p}_3}^{\mu\nu}\Bigg].\end{aligned} \quad (2)$$

The $n(n-1)/2$ component fields are symmetric in the replica indices and $\phi^{\alpha\alpha}\equiv 0$; the spin-glass limit requires $n\to 0$. The total number of spins N is included to ensure the correct thermodynamic limit, and momentum conservation is understood in the primed summation. The zero-field paramagnetic phase corresponds to higher symmetry,[15] with all the bare parameters but with m_1 and w_1 equal to zero; it has a unique mass Γ, and below the upper critical dimension $d_u=6$ the spin-glass transition is governed by the fixed point[16] $w_1^{*2}\equiv w^{*2}=\epsilon/(2-n)$ and $m_1^*=-\epsilon/2$, $\epsilon=6-d$. The mass is split by an external magnetic field into the three different components Γ_R, Γ_A, and Γ_L, thus generating—while replica symmetry is still preserved—a kind of quadratic symmetry breaking. The crossover region is best studied by the introduction of the nonlinear scaling fields[17] satisfying the *exact* renormalization flows $\dot{g}_i=\lambda_i g_i$. [The λ_i's of the mass sector ($i=1,2,3$) were computed from the renormalization flow equations, in leading order, in Ref. 18.] The RG equations provide a way to express the bare parameters of the Lagrangian in terms of the scaling fields; hence the masses can be computed as functions of the g_i's,

$$\Gamma_R=g_1+2g_2+g_3+O(\epsilon),$$

$$\Gamma_A = g_1 - (n-4)g_2 - (n-3)g_3 + O(\epsilon),$$

$$\Gamma_L = g_1 - 2(n-2)g_2 + \frac{(n-2)(n-3)}{2}g_3 + O(\epsilon). \quad (3)$$

The $O(\epsilon)$ terms neglected above have two contributions: the one-loop self-energy (which is computable at that order) and corrections to the bare masses expressed in terms of the g_i's—this, however, is not available in a leading-order RG calculation. Couplinglike scaling fields g_i's with $i>3$ enter also at this $O(\epsilon)$ level. Equation (3) can be derived by fixing the bare parameters such that $\langle \phi_{\mathbf{p}}^{\alpha\beta}\rangle \equiv 0$; this condition determines g_0 unambiguously in terms of the other g_i's. Two critical surfaces can be found from Eq. (3) in the low-temperature ($g_1<0$) regime: (i) $\Gamma_R=0$ and Γ_A and Γ_L are both positive—i.e., an Almeida-Thouless instability—for $g_2<-g_1$ and (ii) $\Gamma_A=0$ and Γ_L and Γ_R are positive ($n\gtrsim 0$) for $g_2>-g_1$. The common boundary of these two manifolds (which are two dimensions now, but allowing for couplinglike scaling fields g_i, $i>3$, they will have a complicated higher dimensional structure) for $g_2=-g_1$ is massive only in the longitudinal sector.[11]

We are now interested in the RG flows along the AT instability surface when starting in the crossover region. The first-order RG equations were all presented in Ref. 18; their structure is best displayed by the following (temporary) redefinition of the couplings: $w_i/\sqrt{\epsilon} \to w_i$, which are now, like the masses, order unity. With the scaling factor e^{dl} and $t \equiv \epsilon l$,

$$\frac{dm_i}{dl} = 2m_i - \epsilon \mathcal{M}_i(m_1,m_2,m_3;w_1,\ldots,w_8), \quad i=1,2,3, \quad (4)$$

$$\frac{dw_i}{dt} = \frac{1}{2}w_i + \mathcal{W}_i(m_1,m_2,m_3;w_1,\ldots,w_8), \quad i=1,\ldots,8. \quad (5)$$

The $\mathcal{M}_i$ and $\mathcal{W}_i$ functions are quadratic and cubic, respectively, in the couplings. The most important feature of the RG equations above is that the flow parameter l in the mass sector [Eq. (4)] is much larger for $\epsilon \ll 1$ than t of the couplings [Eq. (5)]. Thus the masses renormalize in the background of the adiabatically slow couplings: the anomalous (A) and longitudinal (L) components, as they are $O(1)$ on the AT surface, blow up, whereas the replicon (R) one, $2m_1=O(\epsilon)$, evolves into its adiabatic fixed point determined by the initial values of the couplings w_1^{0+} and w_2^{0+}. While $w_1^{0+}=w_1(t=0)=w_1^*$, we must carefully follow the development of w_2 in the transient regime from $w_2(t=0)=0$ (Ref. 21) to $w_2^{0+}\equiv w_2(t)$ with $\epsilon \ll t \ll 1$ for the following reason: for $l\gg 1$, i.e., $t\gg\epsilon$, w_1 and w_2 decouple from the other bare parameters, and their flow can be put into the pair of equations

$$\frac{dw_1}{dt} = \frac{1}{2}w_1 + g_n(r)w_1^3,$$

$$\frac{dr}{dt} = -h_n(r)w_1^2,$$

where $g_n(r)$ and $h_n(r)$ are cubic and quartic polynomials of r with coefficients which are simple polynomials of n and $r\equiv w_2/w_1$. For the case $n=0$, these equations were derived and discussed in Ref. 10. We are now interested in the more generic case $0\le n \lesssim \epsilon$ and observe that the qualitative behavior of the renormalization flow changes drastically when the initial value of $r(0)=w_2^{0+}/w_1^{0+}$ passes through $r_1^*=\frac{3}{10}n+O(n^2)$, with the unstable fixed point r_1^* being the solution of the equation $h_n(r)=0$. For $r(0)>r_1^*$, we have runaway trajectories already noticed in Ref. 10 with $w_1\to\infty$ and $r\to r_2^*\cong 14.4+O(n)$; whereas for $r(0)<r_1^*$, w_2 immediately becomes negative, which is physically nonsense.

To get $r(0)$, we must integrate Eq. (5) for $i=2$ in the transient regime from $t=0$ to $\epsilon\ll t\ll 1$, thereby eliminating nonreplicon modes in the process of hardening anomalous and longitudinal masses. This is feasible using Eq. (65) of Ref. 18 together with the table between the different sets of couplings in Eq. (49) of Ref. 14, resulting in $r(0)<r_1^*$ for $0<n\ll 1$ and starting close enough to the zero-field fixed point, whereas the spin-glass case is exceptional with the condition $r(0)>r_1^*$ *always* fulfilled.

Runaway flows along critical surfaces have been associated with first-order transitions in some common situations with crossover phenomena,[19] and although this scenario cannot be ruled out completely for the spin glass either, we will argue that renormalization of the bare couplings on the AT surface toward their *low-temperature* limit may cause the runaway trajectories in this renormalization scheme. To see this, we recall the derivation of the microscopic Lagrangian in Ref. 14 and the necessity to redefine the fields as $c\phi_{\mathbf{p}}^{\alpha\beta}\to\phi_{\mathbf{p}}^{\alpha\beta}$, with $c\sim T$, to ensure the proper normalization of the kinetic term in $\mathcal{L}^{(2)}$. This will cause the couplings to diverge even if they disappeared for $T\to 0$ otherwise. That kind of normalization was essential in the derivation of Eqs. (4) and (5), manifested in the flowing η exponents of the three different mass modes. As our approximate RG equations are valid only for $w_i=O(1)$, one probably needs to modify the RG scheme for detecting the proper zero-temperature behavior on the AT surface in this small ϵ regime. This is, however, out of the scope of the present work.

In the remaining part of this Rapid Communication we want to locate the AT line of the original Edwards-Anderson spin-glass model[1] on the AT surface of the field theory above. For this reason, we must find out the dependence of the bare parameters in Eqs. (1) and (2) on temperature (T) and magnetic field (H). The criterion which is adopted here is that the tree approximation of the field theory (i.e., neglecting loops) be equivalent with the accepted mean-field theory of the Ising spin glass, the SK model, whose replicated partition function has the form[2]

$$\overline{Z^n} \sim \int \mathcal{D}q \exp\left\{-N\left[\frac{(kT)^2}{2J^2}\sum_{\alpha<\beta} q_{\alpha\beta}^2 - \ln\zeta\right]\right\}, \quad (6)$$

where

$$\zeta = \underset{\{S^\alpha\}}{\mathrm{Tr}} \exp\left(\sum_{\alpha<\beta} q_{\alpha\beta} S^\alpha S^\beta + \frac{H}{kT} \sum_\alpha S^\alpha \right), \tag{7}$$

$\int \mathcal{D}q \equiv \Pi_{\alpha<\beta}(\int N^{1/2} \frac{kT}{\sqrt{2\pi J^2}} dq_{\alpha\beta})$, and J^2, the variation in the Gaussian distribution of the random Ising interactions, sets the energy scale. In the tree approximation fluctuations are omitted, which can be achieved by setting $\phi^{\alpha\beta}_{\mathbf{p}=0} = \sqrt{N} q_{\alpha\beta}$ and zero for $\phi^{\alpha\beta}_{\mathbf{p}\neq 0}$ in Eqs. (1) and (2) and comparing it with Eqs. (6) and (7). Not forgetting that the bare parameters of the field theory are finally tuned by the transformation $\phi^{\alpha\beta}_{\mathbf{p}} - \sqrt{N} q \delta^{\mathrm{Kr}}_{\mathbf{p}=0} \to \phi^{\alpha\beta}_{\mathbf{p}}$—rendering the one-point function to zero—where q is the exact replica symmetric order parameter, they are expressed by T and H in the vicinity of the mean-field critical point $kT^{\mathrm{mf}}_c = J$ as follows:

$$wh = \frac{1}{2}(H/kT)^2 - (m_{1c} - \tau)(wq) + \frac{1}{2}(n-2)(wq)^2 + \cdots,$$

$$m_1 = (m_{1c} - \tau) + (wq) + (H/kT)^2 - \frac{1}{2}q^2\left[u_{01} + u_{02} + \frac{1}{3}(n-1)u_{03} + \frac{1}{3}n(n-1)u_{04}\right]\cdots,$$

$$m_2 = -(wq) - (H/kT)^2 - \frac{1}{3}q^2[(n-3)u_{01} + u_{03}] + \cdots,$$

$$m_3 = -\frac{1}{6}q^2[u_{01} + 2u_{04}] + \cdots,$$

$$w_1 = w + O(q,H^2), \quad w_i = O(q,H^2), \quad i = 2, \ldots, 8. \tag{8}$$

Neglected terms above are higher orders in q and H^2. The quartic couplings are not included here, although their calculation is similarly straightforward, and they may be important above eight dimensions. $\tau > 0$ measures the distance from the critical temperature of the field theory (T_c), whereas $m_{1c} = -\frac{k^2}{2J^2}(T^{\mathrm{mf}2}_c - T^2_c)$ gives the shift in the critical temperature, and is therefore one-loop order. The field theory is defined by τ, $(H/kT)^2$ and by the bare parameters of the symmetrical theory (zero magnetic-field paramagnet): w (cubic coupling), u_{01}, u_{02}, u_{03}, u_{04} (quartic couplings), etc. (see Ref. 20 for the classification of the quartic couplings). To reproduce the SK model results in the tree approximation, we must set $w=1$, $u_{01}=3$, $u_{02}=2$, $u_{03}=-6$, and $u_{04}=0$.

The condition $\langle \phi^{\alpha\beta}_{\mathbf{p}} \rangle = 0$ provides us the equation of state, i.e., the order parameter q around T_c; it is used here to eliminate τ from our results, replacing it by q. The calculation of the one-loop contribution to the equation of state and to the replicon mass is somewhat lengthy due to the complicated replica structure even in the case of replica symmetry. Nevertheless, it is still feasible by the methods in Ref. 14. The result valid for $d>8$, including the SK model by simply taking $d=\infty$, can be put into the scaling form

$$\Gamma_R = (wq)^2 \tilde{\Gamma}_R(x,y), \quad x \equiv \frac{(H/kT)^2}{(wq)^3}, \quad y \equiv \frac{n}{(wq)}. \tag{9}$$

The scaling function $\tilde{\Gamma}_R$ has the simple linear form

$$\tilde{\Gamma}_R(x,y) = ax + by + c, \quad d > 6, \tag{10}$$

with a and b analytical down to four and six dimensions, respectively, and having their loop expansions in terms of $I_k \equiv \frac{1}{N}\sum^\Lambda \frac{1}{p^k}$,

$$a = 1 - 2w^2 I_4,$$

$$b = 1 - 2w^2 I_6 + (-u_{10} + u_{30} + 4u_{40}) I_4, \quad d > 6. \tag{11}$$

However, c blows up at eight dimensions due to the infrared divergence developing in the first-order contribution behind the mean-field term,

$$c = -\frac{2}{3}u_{20}w^{-2} - 16w^2 I_8 + \text{terms with } I_6 \text{ and } I_4, \quad d > 8. \tag{12}$$

As a result, scaling of the replicon mass turns to the following form when $6<d<8$:

$$\Gamma_R = (wq)^{d/2-2}\tilde{\Gamma}_R(x,y), \tag{13}$$

with $x \equiv (H/kT)^2/(wq)^{d/2-1}$ and $y \equiv n/(wq)^{d/2-3}$. The scaling function preserves form (10) with a and b in Eq. (11); the constant c, however, becomes, instead of Eq. (12),

$$c = -16w^2 \int^\infty \frac{d^d p}{(2\pi)^d} \frac{1}{p^4(p^2+2)^2}, \quad 6 < d < 8. \tag{14}$$

The zeros of the scaling function provide the AT transitions, and two important cases can be studied for $d>6$: (i) $H=0$, i.e., $x=0$ and $y=-c/b$ (this case has been discussed in Ref. 20) and (ii) the spin-glass limit $n=0$, i.e., $y=0$ and $x = x_0 = -c/a$. The AT line close to T_c in the two regimes is

$$(H/kT)^2 = x_0 (wq)^3, \quad 8 < d,$$

$$(H/kT)^2 = x_0 (wq)^{d/2-1}, \quad 6 < d < 8. \tag{15}$$

From Eqs. (11) and (12), with $u_{20}=2$, the SK value $x_0 = 4/3$ is reproduced, whereas x_0 becomes one-loop order for $6 < d < 8$ [see Eq. (14)].

Below six dimensions, the leading scaling behavior can be obtained by using fixed-point values in Eq. (8) and also neglecting correction terms provides

$$w^* h = \frac{1}{2}(H/kT)^2 - (m^*_1 - \tau)(w^* q) + \frac{1}{2}(n-2)(w^* q)^2,$$

$$m_1 = m^*_1 - \tau + (w^* q), \quad m_2 = -(w^* q), \quad m_3 = 0,$$

$$w_1 = w^*, \quad w_i = 0, \quad i = 2, \ldots, 8.$$

After eliminating τ by the equation of state, we are left with a two-parameter theory, with the simple RG flows close to the fixed point,

$$\dot{q} \simeq (2 - \epsilon/2 + \eta^*/2) q, \quad (H/\dot{k}T)^2 \simeq \lambda_0 (H/kT)^2, \tag{16}$$

with $\lambda_0 = 4 - \epsilon/2 - \eta^*/2$ and $\eta^* = -\epsilon/3$. The scaling fields can now be expressed as

$$g_i \simeq (w^* q)^{z_i} \tilde{g}_i(x),$$

$$x = \frac{(H/kT)^2}{(w^* q)^\delta}, \quad \delta = \frac{4 - \epsilon/2 - \eta^*/2}{2 - \epsilon/2 + \eta^*/2}.$$

From Eq. (16) it follows that x is invariant under renormalization and z_i is equal to $(2-\epsilon/2+\eta^*/2)^{-1}\lambda_i$ for $\dot{g}_i = \lambda_i g_i$ must be satisfied. Any observable $\mathcal{O}$ satisfying the approximate RG flows $\dot{\mathcal{O}} \simeq k_{\mathcal{O}} \mathcal{O}$ around the fixed point can now be written as $\mathcal{O} \sim (w^* q)^{k_{\mathcal{O}}/(2-\epsilon/2+\eta^*/2)}$ times a function of x. For a mass $k = 2 - \eta^* = (\delta - 1)(2 - \epsilon/2 + \eta^*/2)$, and therefore the replicon mass takes the scaling form

$$\Gamma_R \simeq (w^* q)^{\delta - 1} \tilde{\Gamma}_R(x), \quad x = \frac{(H/kT)^2}{(w^* q)^\delta}. \qquad (17)$$

The most important feature of Eq. (17) when compared with the $d > 6$ cases [Eqs. (9) and (13)] is the lack of the second scaling variable, which is proportional to n. The AT line ends now in the zero-field critical point[22] even for small n but nonzero,

$$(H/kT)^2 = x_0 (w^* q)^\delta, \quad x_0 = -n + [2 + O(n)]\epsilon + O(\epsilon^2), \qquad (18)$$

and it disappears completely for $n > 2\epsilon$.

To conclude, we followed the fate of the AT line from mean field down to $d = 6 - \epsilon$. (See Fig. 1 for the cases just above and below six dimensions.) An exceptional feature of the spin-glass case ($n = 0$) is that the runaway flows toward zero-temperature behavior—found below $d = 6$—originate in the close vicinity of the zero-field fixed point. Our results do not exclude a possible lack of the AT surface in $d = 3$—as suggested by recent numerical works[6,7]—a scenario, with some lower critical dimension to explain this, that has been suggested in Ref. 20.

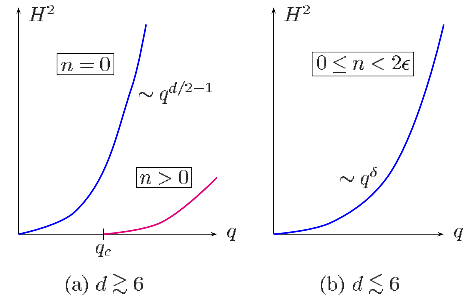

FIG. 1. (Color online) AT lines on the two sides of the upper critical six dimensions [see Eqs. (15) and (18) and footnote (Ref. 22)].

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[21] More precisely, $w_2(t=0) \ll \epsilon$ in the physically relevant part of the AT surface.

[22] When $6 < d < 8$ the AT line takes the form $(H/kT)^2 \sim n^{1-2/\epsilon} (q - q_c)$ for $n \gtrsim 0$, where $q_c \sim n^{-2/\epsilon}$. For $d \geq 8$ the mean-field phase diagram restores (Refs. 9, 14, and 20).